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SCALING LAW OF VOLCANIC EXPLOSION EARTHQUAKE

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Abstract. The aim of the present study is to clarify a scaling law for volcanic explosion earthquakes excited by a counter force of eruption. Based on the data of explosion earthquakes at three volcanoes, we derived the following results: The peak amplitude of the single force is proportional to the square of the pulse width of source time function. The initial pressure stored in a shallow region beneath a volcano before eruption is 1 MPa with a perturbation of one order. The kinetic energy of explosion earthquake is proportional to the cube of crater radius. The scaling law of the explosion earthquake is quite similar to that of the earthquake caused by a faulting. It is possible to predict the strength of explosion earthquake by measuring the crater radius beforehand.

Introduction

Volcanic explosion earthquakes accompany individual explosive eruption. A comprehensive study of the source process of the explosion earthquake is important not only to clarify the dynamics of eruption but also to advance the prediction of volcanic eruption. Many researchers have, therefore, studied the source mechanism of explosion earthquakes observed at many volcanoes in the world.

Kanamori and Given (1983) analyzed the long-period seismograms of the explosion earthquakes associated with the 1980 eruptions of Mt. St. Helens. They first showed that the source mechanism of an explosion earthquake can be expressed by a vertical downward single force. Kanamori et al. (1984) further proved that the force system of volcanic eruption is equivalent to a single force that represents a counter force of eruption. Takeo et al. (1984) also showed that the seismograms of radial and vertical components of an explosion earthquake observed at Mt. Asama, Japan, can be explained by multiple single forces. Recently, Nishimura (1991) analyzed the volcanic earthquakes accompanied with the 1988-89 eruptive activities of Mt. Tokachi, Japan, and showed that main phases of not only explosion earthquakes but also low-frequency earthquakes can be interpreted as the waves excited by counter forces of eruption.

These previous studies show that the source mechanism of explosion earthquake can be interpreted by a single force that represents the counter force of eruption. However, physical basis on the magnitude of the force system remains still unresolved, in spite of its importance for evaluating the magnitude of explosion earthquake.

The purpose of the present study is to clarify the physical basis of the magnitude of explosion earthquake. There-

fore, we revise the single force model of Kanamori et al. (1984) based on reasonable assumptions for a shape of a reservoir and a flow of volcanic ejecta in the next section. Subsequently, we show a systematic relation between the peak amplitude and pulse width of the single force estimated from four explosion earthquakes. Finally, we propose a scaling law of explosion earthquakes, and discuss on its source parameters.

Single Force Model

Kanamori et al. (1984) considered that the volcanic eruption is dynamically a release process of pressure that has been reserved inside a shallow region just beneath a crater. They assumed a reservoir of cylindrical shape with a radius of r and a depth of $2r$ (see Figure 1(a)), and showed that the seismic waves associated with a volcanic eruption are predominantly excited by a vertical downward single force acting on the bottom of the reservoir. Then, the peak amplitude of the single force is given by

$$F = \pi r^2 P_0 \quad (1)$$

where P_0 is the pressure in the reservoir before eruption.

We further consider the behavior of the flow of volcanic ejecta associated with eruption. For simplicity, we assume that the flow of volcanic ejecta can be expressed by an isentropic and perfect gas, though the flow consists of solids, liquids, and gasses. Once an eruption occurs, the flow accelerates from the reservoir to the upper side. When the exit of the reservoir is a throat and the pressure in the reservoir is about two times higher than that of atmosphere, the flow reaches the sonic (choked) conditions in the exit of the reservoir that plays a nozzle [Kieffer, 1989]. The temporal variation of pressure can be expressed by [e.g., Kieffer and Sturtevant, 1984]

$$p(t) = P_0 \left(1 + \frac{\kappa - 1}{2} G t \right)^{-2\kappa/(\kappa - 1)} \quad (2)$$

where

$$G = \nu_0 \frac{S}{V} \left(\frac{2}{\kappa + 1} \right)^{(\kappa + 1)/2(\kappa - 1)} \quad (3)$$

and, ν_0 is the initial particle velocity of the flow, V is the

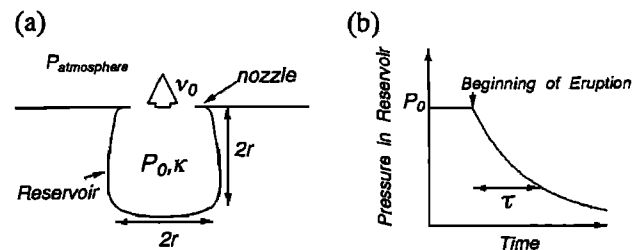


Fig. 1. (a) A pressure release model for a volcanic eruption. The exit of reservoir plays a role of nozzle.

(b) Temporal variation of pressure in the reservoir.

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Table 1. Peak amplitude F and pulse width τ of the single forces estimated from four explosion earthquakes.

No.	Volcano	Date/Time(GMT)	$F(N)$	$\tau(sec)$	Reference
1	St. Helens	1980/05/18 15:32	2.6×10^{12}	25	Kanamori et al.(1984)
2	Asama	1983/04/07 16:59	1.5×10^{10}	4	This Study
3	Tokachi	1989/02/07 19:02	5.0×10^9	1	Ohshima et al.(1989)
4	Tokachi	1989/01/27 20:18	4.0×10^9	0.6	This Study

volume of the reservoir ($= 2\pi r^3$), S is the area of crater ($= \pi r^2$), and κ is the specific heat of gas in the reservoir. The pressure in the reservoir decreases almost exponentially with increasing a time, because the specific heat κ usually takes a value ranging from 1.1 to 1.4 [Kieffer, 1989]. Since the single force applied to the volcano decreases with the pressure inside the reservoir [Kanamori et al., 1984], we define the pulse width of the single force as duration time that the pressure decreases to $1/e$ of P_0 :

$$\tau = c_1 \frac{\tau}{\nu_0} \quad (4)$$

where

$$c_1 = \frac{4}{\kappa - 1} \left(\frac{\kappa + 1}{2} \right)^{(\kappa+1)/(2(\kappa-1))} (e^{(\kappa-1)/2\kappa} - 1) \quad (5)$$

Here, c_1 is estimated to be 3 ± 0.3 , by assuming $\kappa = 1.1$ to 1.4.

Eliminating r in eqs.(1) and (4) yields

$$\log F = 2 \log \tau + g(\nu_0, P_0) \quad (6)$$

where

$$g(\nu_0, P_0) = 2 \log(\nu_0/c_1) + \log P_0 + \log \pi \quad (7)$$

Consequently, it is interesting to examine an actual relation between the peak amplitude F and the pulse width τ as shown in the next section.

Data and Analysis

In Table 1, the value of F and τ are listed for four events obtained at three volcanoes, Mt. St. Helens (U.S.A; 46.20° N, 122.18° W), Mt. Asama (Japan; 36.40° N, 138.53° E), and Mt. Tokachi (Japan; 43.42° N, 142.68° E). Among them, events 2 and 4 are analyzed in the present study based on the long-period seismograms of WWSSN at Matsushiro station (MAT; epicentral distance $\Delta=32$ km), and velocity-type seismometers with a natural frequency of 0.2 Hz at Tokachi station (TKC; $\Delta=2.1$ km; Nishimura et al., 1990). In order to get the values of F and τ , observed records are compared with theoretical seismograms for these events using a discrete wave number method [Bouchon, 1979] with assumption of a source time function of triangular type. In this calculation, according to Takeo et al.(1984) and Nishimura (1991), we assumed the source depths of events 2 and event 4 to be 1 km and 0.2 km and the velocity models (Table 2). Figure 2 shows the best-fit theoretical seismograms with the observed ones for event 2 and 4.

Figure 3 shows a linear relation between F and τ , suggesting a regression line of

$$\log F = 1.7 \log \tau + 9.7 \quad (8)$$

with a strong correlation coefficient of 0.95. The result present a good agreement with a theoretical relation of eq.(6). Based on eq.(6), if we assume the slope of $\log \tau$ as 2, a regression line is obtained as follows:

$$\log F = 2.0 \log \tau + (9.6 \pm 0.6) \quad (9)$$

or

$$g(\nu_0, P_0) = 9.6 \pm 0.6 \quad (10)$$

For estimating the initial pressure P_0 , we will discuss the particle velocity of volcanic ejecta associated with explosive eruptions. Associated with the crimatic eruption of Mt. St. Helens on May 18, 1980, the maximum velocity of summit plume was estimated at 90 m/s, based on successive photo-pictures [Voight, 1981]. The initial velocities ν_0 of erupted rocks at the 1988-89 eruptive activities of Mt. Tokachi were estimated at 70 to 120 m/s [Sato et al., 1990]. At Sakurajima Volcano, which is well known as a high activity of explosive eruption, ν_0 was around 110-160 m/s [Iguchi et al., 1983]. If so, $\log P_0$ is obtained as 6.1 ± 1.1 from eqs.(7) and (10) with assumption of $c_1 = 3 \pm 0.3$. In other words, the initial pressure P_0 takes the value of 1 MPa with a perturbation of one order.

Discussion

It is widely known that seismic energy E_s of a tectonic earthquake, which is caused by a faulting, can be expressed by

$$E_s = c_2 L^3 \sigma \quad (11)$$

where L is the fault length, σ is the stress drop and c_2 is a constant. Since the stress drop of tectonic earthquakes takes a constant value ranging from the order of 1 to 10 MPa [Kanamori and Anderson, 1975], the seismic energy is mainly scaled by the fault length.

Table 2. Velocity structures for events 2 and 4.

Layer	V_p (km/s)	V_s (km/s)	ρ (g/cm ³)	Thickness (km)
(event 2)				
1	6.0	3.5	2.5	∞
(event 4)				
1	2.8	1.6	1.8	0.1
2	3.4	2.0	2.0	0.5
3	4.1	2.4	2.2	0.5
4	4.7	2.7	2.4	∞

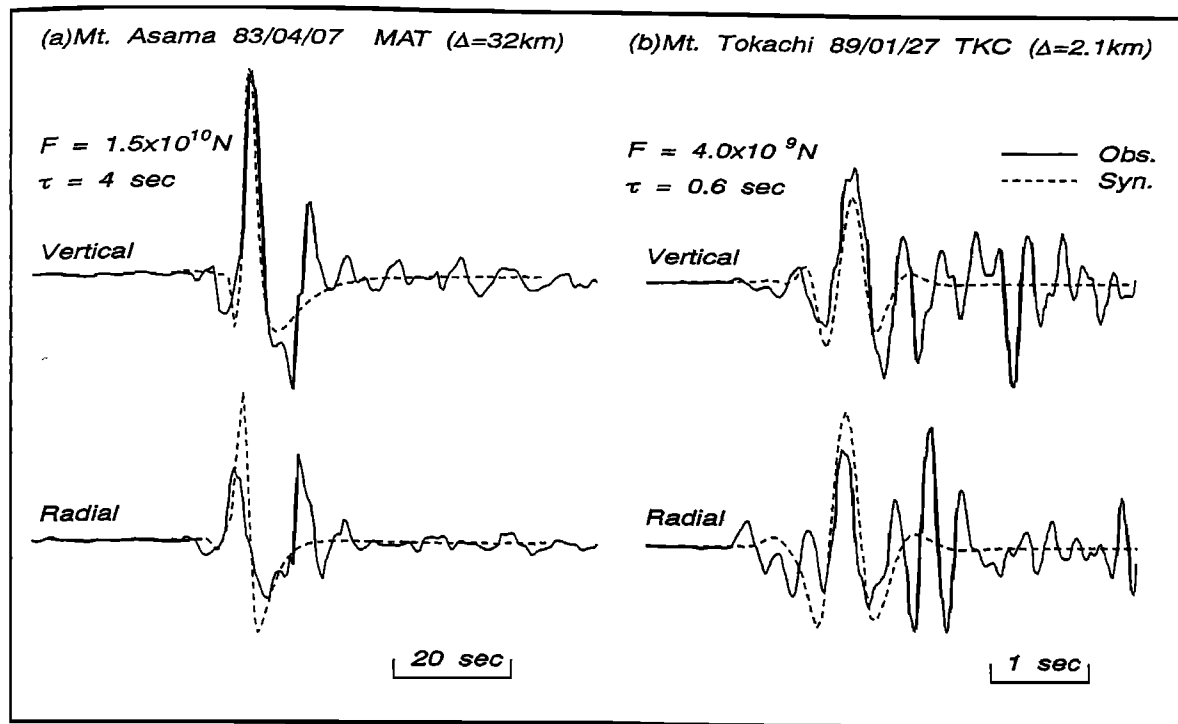


Fig. 2. Comparison between the observed (solid curves) and the synthetic (broken curves) seismograms of explosion earthquakes for (a) event 2 at Mt. Asama (Japan) and (b) event 4 at Mt. Tokachi (Japan)

As for the explosion earthquake, we can define the kinetic energy of ejecta E_k by

$$E_k = \int_0^\infty \frac{1}{2} \rho_* \nu_*^3 S dt \quad (12)$$

where ρ_* and ν_* are the density and velocity of the flow, respectively, at the exit of a reservoir having an area of

S . Using the isentropic flow relations and the equation of perfect gas, we can derive

$$E_k = c_3 r^3 P_0 \quad (13)$$

where

$$c_3 = \frac{1}{2} \left(\frac{2}{\kappa + 1} \right)^{2\kappa/(\kappa-1)} \quad (14)$$

This simple equation implies that the crater radius r mainly control the magnitude of E_k , because P_0 has almost a constant value as discussed in the previous section.

We can find a good similarity between eq.(11) and eq.(13): The crater radius r corresponds to the fault length L and the constant pressure P_0 does to the constant stress drop σ . In a word, the scaling law of explosion earthquake is quite similar to that of the tectonic earthquake.

The scaling law motivates us to estimate the magnitude of an explosion earthquake from the actual crater radius. We measured the crater radii of the three volcanoes from a topographic map, and estimated the value of F and τ from eqs.(1) and (4), respectively, assuming that $P_0 = 1$ MPa and $\nu_0 = 100$ m/s. We call them as the predicted F and τ in the followings. Table 3 summarizes

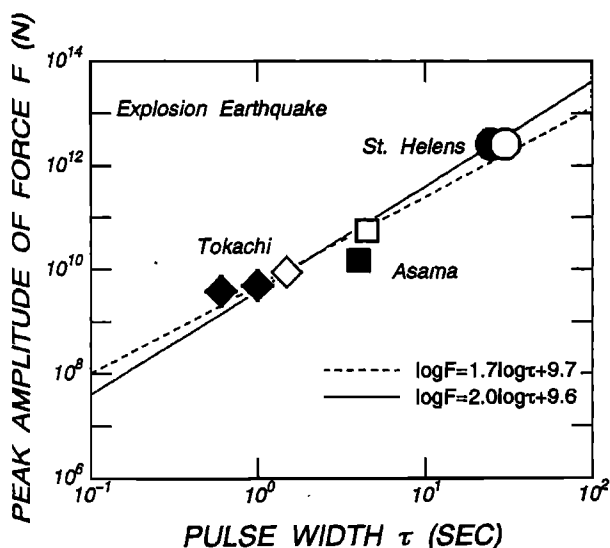


Fig. 3. Relation between the peak amplitude F and the pulse width τ of the single force. The observed and the predicted ones are plotted by solid and open symbols, respectively.

Table 3. Crater radius of volcano and the predicted values of peak amplitude F and pulse width τ .

Volcano	Crater radius(m)	F (N)	τ (sec)
St. Helens	1000	2.6×10^{12}	30
Asama	150	5.6×10^{10}	4.5
Tokachi	50	9.0×10^9	1.5

the crater radii measured from topographic maps and the predicted F and τ of Mt. St. Helens, Mt. Asama, and Mt. Tokachi. As for Mt. St. Helens, we adopted the average radius of major and minor axes of the horseshoe-shaped crater formed just after the main eruption on May 18, 1980. We measured the radii of pre-existing crater for Mt. Asama and Mt. Tokachi so that the predicted F and τ of them are expected to be the maximum values. Figure 3 plots the predicted F and τ for each volcano by open symbols. We see that the predicted F and τ for each volcano are quite close to the observed ones. This good agreement suggests that prediction of the magnitude of explosion earthquake is possible by measuring the crater radius beforehand. For the case of Mt. St. Helens, the size of the present horseshoe-shaped crater was not known before the eruption. However, the epicenters of small earthquakes during the two months before the occurrence of main eruption were located within a small area having a radius of about 1500 m. This area corresponds to the horseshoe-shaped crater formed by the main eruption [see fig.56 in Endo et al.(1981)]. Hence, if we had been able to clarify the relation between the newly formed crater and the size of the epicentral regions, we could predict the magnitude of the explosion earthquake before the occurrence of the main eruption.

Conclusion

Based on a single force model, which represent a counter force of volcanic explosion, a scaling law of volcanic explosion earthquakes are obtained as follows:

- (1) The peak amplitude of single force is proportional to the pulse width of the force.
- (2) The kinetic energy associated with the explosion earthquake is proportional to the cube of crater radius.
- (3) The initial pressure in the reservoir associated with explosion earthquake takes a constant value of 1 MPa with a perturbation of one order.
- (4) The scaling law of explosion earthquake is quite similar to that of tectonic earthquake.
- (5) It is possible to predict the strength of explosion earthquake by measuring the crater radius beforehand.

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